Name _____ ID3 _ _ _

**ECS 315: Probability and Random Processes** **2019/1**

# HW 10 — Due: November 14, 4 PM

*Lecturer: Prapun Suksompong, Ph.D.*

## Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers **directly on the provided hardcopy/file** (not on other blank sheet(s) of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: "5565242231 315 HW10.pdf"

(d) (8 pt) It is important that you try to solve all problems.

(e) Late submission will be heavily penalized.

**Problem 1** (Yates and Goodman, 2005, Q3.2.1)**.** The random variable $X$ has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find the following quantities.

(a) the constant $c$

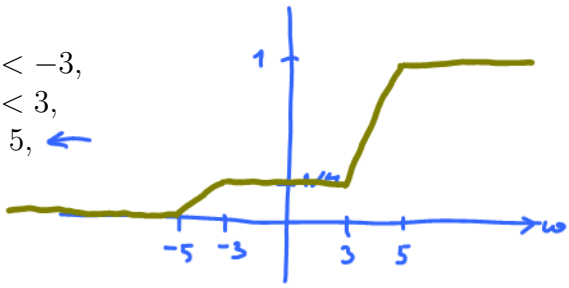
(b) $P[0 \le X \le 1]$

(c) $P[-1/2 \leq X \leq 1/2]$.

(d) the cdf $F_X(x)$.

**Problem 2** (Modified from Yates and Goodman, 2005, Q3.1.3)**.** The CDF of a random variable $W$ is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3(w-3)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$




(a) Is $W$ a continuous random variable?

Yes because $F_W(w)$ is continuous everywhere.

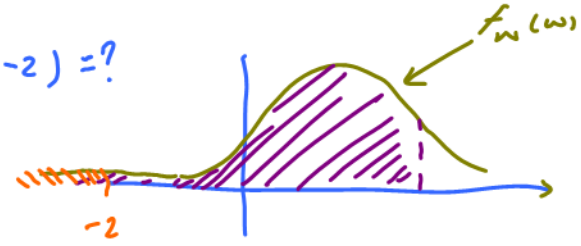
(b) What is $P[W \leq 4]$?

$F_W(4) = \frac{1}{4} + 3\left(\frac{4-3}{8}\right) = \frac{5}{8}$

(c) What is $P[-2 \leq W \leq 2]$? $= F_W(2) - F_W(-2) = ?$

$F_W(2) = \int_{-\infty}^{2} f_W(w)\,dw$

$F_W(-2) = \int_{-\infty}^{-2} f_W(w)\,dw$




(d) What is $P[W > 0]$?

(e) What is the value of $a$ such that $P[W \leq a] = 1/2$?

**Problem 3** (Yates and Goodman, 2005, Q3.2.3)**.** The CDF of random variable $W$ is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3(w-3)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$

Find its pdf $f_W(w)$.

$$f_W(w) = \frac{d}{dw} F_W(w) = \begin{cases} 0, & w < -5, \\ 1/8, & -5 < w < -3, \\ 0, & -3 < w < 3, \\ 3/8, & 3 < w < 5, \\ 0, & w > 5 \end{cases} \Rightarrow f_W(w) = \begin{cases} 1/8, & -5 < w < -3, \\ 3/8, & 3 < w < 5, \\ 0, & \text{otherwise.} \end{cases}$$

**Problem 4** (Yates and Goodman, 2005, Q3.3.4)**.** The pdf of a cont. random variable $Y$ is

$$f_Y(y) = \begin{cases} y/2 & 0 \leq y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbb{E}[Y]$.

$$= \int_{-\infty}^{\infty} y f_Y(y)\, dy$$

(b) Find $\operatorname{Var} Y$.

**Problem 5** (Yates and Goodman, 2005, Q3.3.6)**.** The cdf of random variable $V$ is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v+5)^2/144, & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is $f_V(v)$?

(b) What is $\mathbb{E}[V]$?

(c) What is $\mathrm{Var}[V]$?

(d) What is $\mathbb{E}[V^3]$?